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LETTER TO THE EDITOR

Non-local potentials and positive energy bound states

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Abstract. For a non-local separable potential it is shown that, in general, a positive energy bound state cannot be interpreted as a zero-width resonance.

Positive energy bound states (PEBS) have been interpreted as resonances of zero-width (Bolsterli 1969, Beam 1969). The interpretation is based on the solution of the Schrödinger equation with a rank-one separable potential. We will show that a rank-two separable potential can give a PEBS and a scattering phase shift at the same energy.

Consider the s-wave Schrödinger equation

$$(\mathrm{d}^2/\mathrm{d}r^2 + k^2)\psi(r) = \int_0^\infty K(r, r')\psi(r')\,\mathrm{d}r' \tag{1}$$

with

$$K(r, r') = \sum_{i=1}^{2} \lambda_i g_i(r) g_i(r') \tag{2}$$

$$g_i(r) = \exp(-\beta_i r) \qquad i = 1, 2. \tag{3}$$

The above equations have a solution

$$\psi_b(r) = N[\exp(-\beta_1 r) - \exp(-\beta_2 r)] \tag{4}$$

at the energy $k^2 = k_0^2$ provided that

$$\lambda_1 = 2\beta_1(\beta_1 + \beta_2)(k_0^2 + \beta_1^2)/(\beta_2 - \beta_1) \tag{5}$$

and

$$\lambda_2 = 2\beta_2(\beta_1 + \beta_2)(k_0^2 + \beta_2^2)/(\beta_1 - \beta_2). \tag{6}$$

That is, we have a PEBS at $k^2 = k_0^2$ when the values of λ_1 and λ_2 are given by equations (5) and (6).

Furthermore equations (1), (2) and (3) have the solution (Husain and Ali 1970)

$$\psi_s(r) = \sin kr + \sum_{i=1}^{2} \frac{\lambda_i A_i}{k^2 + \beta_i^2}[\exp(-\beta_i r) - \cos kr] \tag{7}$$

where

$$A_i = \frac{k}{k^2 + \beta_i^2} + \sum_{j=1}^{2} \frac{\lambda_j A_j}{k^2 + \beta_j^2}\left(\frac{1}{\beta_i + \beta_j} - \frac{\beta_i}{k^2 + \beta_i^2}\right) \qquad i = 1, 2 \tag{8}$$

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and therefore

$$\tan\delta = -\sum_{i=1}^{2} \lambda_i A_i/(k^2+\beta_i^2). \tag{9}$$

With the values λ_1, λ_2 given by equations (5) and (6) $\tan\delta$ has a finite value at the energy $k^2=k_0^2$ except when $k_0^2=\beta_1\beta_2$. In that case, coefficients of A_1 and A_2 in equations (8) vanish. The value of $\tan\delta$ at the energy $k^2=k_0^2$ turns out to be

$$\tan\delta = k_0(\beta_1+\beta_2)/(k_0^2-\beta_1\beta_2). \tag{10}$$

Thus we can have a PEBS and scattering at the same energy. So we conclude that, in general, a PEBS cannot be interpreted as a zero-width resonance. The appearance of PEBS in the above example is a property of the separable potential and apparently has no physical meaning. The general conditions under which a positive energy bound state appears as a zero-width resonance will be discussed in a forthcoming publication.

References

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